

Math 128a - Week 5 Worksheet
GSI: Izak, (2/17/21)

2.3 Problems

Problem 1. Derive the error formula for Newton's method:

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|} |p - p_n|^2$$

2.5 Problems

Problem 2. Steffensen's method is applied to a function $g(x)$ using $p_0^{(0)} = 1, p_2^{(0)} = 3$ to obtain $p_0^{(1)} = .75$. What is $p_1^{(0)}$?

2.6 Problems

Problem 3. Use Horner's method to evaluate $P(x) = 7x^4 - 2x^2 - 5x - 3$ at $x = 1$

3.1 Problems

Problem 4. Given $f(x) = x^3 - 4x^2 + 4$, find the Lagrange interpolation polynomial of degree at most three using the nodes $x_0 = -3, x_1 = -1, x_2 = 1, x_3 = 5$

Problem 5. Let $x_0 = -1, x_1 = 0, x_2 = 1$, define $f_0(x) = x^2 - 1, f_1(x) = 2x^2 + 3x, f_2(x) = -x^2 + 2x$. Evaluate these polynomials at x_i . Use this to find a polynomial of degree at most 2 such that $g(x_0) = -4, g(x_1) = -1$, and $g(x_2) = 6$ without performing any tedious computations.

	x_0	x_1	x_2
f_0	-1	0	1
f_1	0	-1	0
f_2	-1	0	5
f_c	-3	0	1

Gauss g is a linear combo.
 $g(x) = a f_0 + b f_1 + c f_2$
 $-4 = g(x_0) = 0 - b - 3c$
 $-1 = -b - 3c$
 $-1 = -a$
 $6 = 5b + c$
 $a = b = c = 1$
 $g(x) = f_0 + f_1 + f_2$

1) Trick Taylor @ $x = p_n$

$$f(x) = f(p_n) + f'(p_n)(x - p_n) + \frac{f''(\xi)}{2}(x - p_n)^2$$

$x = p$

$$0 = f(p) = f(p_n) + f'(p_n)(p - p_n) + \frac{f''(\xi)(p - p_n)^2}{2}$$

$$0 = \frac{f(p_n)}{f'(p_n)} + (p - p_n) + \frac{f''(\xi)(p - p_n)^2}{2 f'(p_n)}$$

$$p_{n+1} := p_n - \frac{f(p_n)}{f'(p_n)}$$

$$\rightarrow 0 = p - p_{n+1} + \frac{f''(\xi)(p - p_n)^2}{2 f'(p_n)}$$



$x = p_1^{(0)}$

2) by def $.75 = 1 - \frac{(x-1)^2}{3-2x+1}$

Solve for x

$x = 0$ and 1.5

3)

$$\begin{array}{c|cccc} 1 & 7 & 0 & -2 & -5 & -3 \\ & \downarrow & & & & \\ & 7 & 7 & 5 & 0 & \\ \hline & 7 & 7 & 5 & 0 & | -3 = P(1) \end{array}$$

$$P(x) = (x-1)(7x^3 + 7x^2 + 5x) - 3$$

$P(1) = -3$
 $P(1) = -3$